

# Coulomb blockade signatures of the topological phase transition in semiconductor-superconductor nanowires

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In semiconductor-superconductor hybrid structures a topological phase transition is expected as a function of chemical potential or magnetic field strength. We show that signatures of this transition can be observed in nonlinear Coulomb blocked transport through a ring shaped structure. In particular, for a fixed electron parity of the ring, the flux periodicity of the neutral excitation spectrum changes from the usual  $h/2e$ -periodicity to a characteristic  $h/e$ -periodicity when tuning the system from the topologically trivial to the nontrivial phase. We relate the  $h/e$ -periodicity to the recently predicted  $4\pi$  periodicity of the Josephson current across a junction formed by two topological superconductors.

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*Introduction.*—The investigation of topological phases of quantum systems has become one of the most exciting developments in the condensed matter community. Of particular interest are the topological properties of wave functions (WFs) and exotic quasiparticles [1, 2]. For this reason, much effort has been invested in the study of topological superconductors (TSCs), which have been predicted to host Majorana fermions [3–10]. One of the defining properties of a topologically ordered state is the degeneracy of the ground state on surfaces with nonzero genus. In particular, the grand canonical ground state of the  $p_x + ip_y$  TSC on the torus strongly depends on the boundary conditions (BCs) for each of the two fundamental cycles [3, 11]. The three ground states with at least one antiperiodic BC are all described by even parity WFs, while the ground state with only periodic BCs shows an odd parity ground-state WF. In contrast, the ordinary  $s$ -wave SC on the torus possesses a four-fold degenerate ground state with an even parity [11].

In this letter, we consider a one-dimensional (1D) TSC on a ring instead of the two-dimensional torus case, and we find that uniquely defined ground states for periodic and antiperiodic BCs show up as a parity effect of the excitation spectrum and lead, for fixed parity, to a doubling of the flux periodicity from the usual  $h/2e$ -period to a characteristic  $h/e$ -period. We show that the excitation spectra of the topological trivial phase are similar to those of  $s$ -wave SCs which have been observed in ultrasmall SC grains [12, 13], whereas we find a strikingly different behavior in the topological nontrivial phase.

One promising candidate for TSCs are semiconducting (SM) nanowires with strong Rashba spin-orbit coupling in a magnetic field and proximity coupled to an  $s$ -wave SC [14–17]. Here, we investigate a ring shaped 1D SM/SC heterostructure pierced by magnetic flux. We consider single-electron tunneling to study the nonlinear Coulomb blocked transport through a weakly coupled

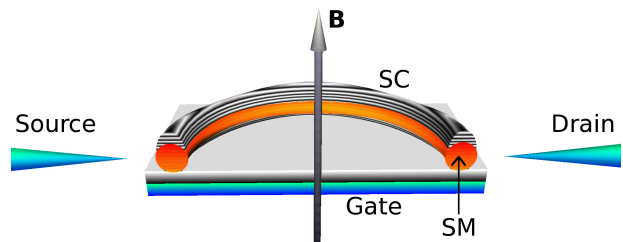


FIG. 1. (Color online) Cut through the experimental setup for a ring shaped SM/SC hybrid system. The SC is sputtered on top of the SM which itself is deposited on a gate electrode.

normal metal-nanowire-normal metal junction. After introducing our model, we discuss its topological nature by comparing it with Kitaev's model [18] for a 1D spinless  $p + ip$  TSC. Using an unbiased numerical minimization for fixed mean particle number and parity, in the limit of zero temperature, we calculate the excitation energies as a function of the magnetic field and flux. The spectra show clear signatures of both the ordinary SC and the TSC phase, and the topological phase transition between them gives rise to the closing and reopening of an excitation gap. Finally, we compare the flux periodicity of the excitation spectra with the  $4\pi$  periodicity of two coupled Majorana chains [18].

*Model system.*—We consider a SM nanowire with strong spin-orbit coupling forming a loop of radius  $R$ , separated from a gate electrode by a thin insulating layer. On top of the nanowire a proximity coupled  $s$ -wave SC is deposited, see Fig. 1. Tunneling into and out of the SM/SC hybrid system is possible via source and drain electrodes. Assuming a strong capacitive coupling between nanowire and SC, the Coulomb energy of the hybrid system is given by

$$H_C = E_C(N + N_{SC})^2 - eV_G(N + N_{SC}), \quad (1)$$

where  $E_C$  denotes the charging energy,  $V_G$  the gate potential, and  $N$  ( $N_{SC}$ ) the number of excess electrons attracted by the gate voltage. The Hamiltonian Eq. (1) describes Coulomb blockade physics of the hybrid system: when the charging energy is degenerate with respect to changing the total electron number  $N + N_{SC}$  by one, a peak in the linear conductance through the hybrid system is observed. For nonzero source-drain voltage  $V$ , resonances in differential conductivity appear when  $eV/2 = E(N \pm 1) - E^{gs}(N)$  where  $E(N)$  is the total energy of an  $N$ -electron state and  $E^{gs}(N)$  the respective ground state energy. The distances between these peaks are independent of the charging energy and directly give the fixed particle number excitation spectrum,  $\delta E(N) = E(N) - E^{gs}(N)$ . We assume that the excitation gap in the SC is much larger than the proximity induced gap in the SM. Then, all electrons in the SC are paired and unpaired electrons can only show up in the SM. In this regime, breaking of Cooper pairs occurs in the SM only and can be observed as resonances in nonlinear Coulomb blockaded conductance, similar to the experiment on metallic nanograins [12].

The Hamiltonian describing the lowest energy subband of the nanowire is given by [19]

$$H_0 = \sum_{k \in \mathbb{Z}} \left\{ \psi_{k\sigma}^\dagger \left[ \frac{\hbar^2}{2m^*R^2} \left( k + \frac{\Phi}{\Phi_0} \right)^2 - \mu + \sigma \frac{g\mu_B B}{2} \right] \psi_{k\sigma} + \frac{\alpha}{R} \left( k + \frac{1}{2} + \frac{\Phi}{\Phi_0} \right) \left( \psi_{k\uparrow}^\dagger \psi_{k+1\downarrow} + \psi_{k+1\downarrow}^\dagger \psi_{k\uparrow} \right) \right\}, \quad (2)$$

where the operator  $\psi_{k\sigma}^\dagger$  ( $\psi_{k\sigma}$ ) creates (annihilates) an electron with spin  $\sigma$  and angular momentum  $\hbar k$ ,  $m^*$  is the effective band mass, and  $\mu$  the chemical potential. We expect the following discussion to hold also for the more general case of an odd number of occupied transverse modes [20]. The Rashba spin-orbit coupling,  $\alpha$ , couples states  $\{|k \uparrow\rangle, |k+1 \downarrow\rangle\}$  and creates two helical bands with the spin rotating within the  $x$ - $y$ -plane. The bands cross each other at  $k = -1/2 - \Phi/\Phi_0$ . The magnetic field,  $B$ , tilts the spin direction out of the  $x$ - $y$ -plane, removes the level crossing, and opens a spin gap  $E_Z = g\mu_B B/2$ .  $\Phi/\Phi_0$  denotes the magnetic flux through the loop in units of the flux quantum  $\Phi_0 = h/e$ . We find the single-particle dispersion of the tilted helical bands,

$$\epsilon_{\pm}(\tilde{k}) = \frac{\hbar^2(\tilde{k}^2 + \frac{1}{4})}{2m^*R^2} - \mu \pm \sqrt{\left( \frac{\hbar^2\tilde{k}}{2m^*R^2} - E_Z \right)^2 + \frac{\alpha^2\tilde{k}^2}{R^2}}, \quad (3)$$

where  $\tilde{k} = k + \Phi/\Phi_0 + 1/2$ .

The  $s$ -wave SC is described within the Ginzburg-Landau formalism by the free energy density

$$f_{GL}[|\Delta_s|, q] = f_0(|\Delta_s|^2) + \frac{\hbar^2|\Delta_s|^2}{2m_sR^2} \left( q + \frac{2\Phi}{\Phi_0} \right)^2 + \frac{B^2}{2\mu_0}, \quad (4)$$

where  $f_0$  is the free energy for zero flux,  $\Delta_s$  the pairing potential,  $\hbar q$  the condensate angular momentum, and  $m_s$  the mass of the Cooper pairs. Minimization of  $f_{GL}$  demands that  $q$  is the integer nearest to  $-2\Phi/\Phi_0$  and that  $\delta f_{GL}/\delta \Delta_s = 0$ . In the following, we neglect the small oscillations in  $|\Delta_s|$  and focus on the large effect of parity and flux on the addition spectrum of the SM ring. The proximity coupling between the  $s$ -wave SC and the nanowire gives rise to a pairing term

$$H_{SC} = \sum_{k \in \mathbb{Z}} \left[ \Delta(\Phi) \psi_{k\uparrow}^\dagger \psi_{-k+q\downarrow}^\dagger + \Delta^*(\Phi) \psi_{-k+q\downarrow} \psi_{k\uparrow} \right], \quad (5)$$

which couples states  $|k \uparrow\rangle$  and  $|-k+q \downarrow\rangle$ . As a consequence, the Hamiltonian is block-diagonal, and within each block a quadruplet  $\{|k \uparrow\rangle, |k+1 \downarrow\rangle, |-k+q \downarrow\rangle, |-k-1+q \uparrow\rangle\}$  is coupled. For odd  $q$ , the quadruplet for  $k = (q-1)/2$  reduces to the doublet  $\{|(q-1)/2 \uparrow\rangle, |(q+1)/2 \downarrow\rangle\}$ . The pairing potential  $\Delta$ , which is reduced in magnitude as compared to the  $s$ -wave potential  $\Delta_s$ , plays a crucial role since it sets two excitation energies. It both opens an effective pairing gap at the Fermi surface and it modifies the Zeeman gap at  $\tilde{k} = 0$ . For  $\Delta^2 > E_Z^2 - \mu^2$  both helicities are occupied in the ground state and  $\Delta$  pairs generalized time-reversed pairs at both sets of Fermi points. Hence, the nanowire is in a topologically trivial state with SC gaps at both  $\pm k_F$  and  $\tilde{k} = 0$ . For  $\Delta^2 < E_Z^2 - \mu^2$  on the other hand, the band structure is different in an important way because now there is a spin gap at  $\tilde{k} = 0$  and an SC gap only at  $\pm \tilde{k}_F$  [16]. If  $E_Z \gg \Delta$ , and if the chemical potential lies well within the spin gap, it is justified to only consider the lower band with energies  $\epsilon_-(k)$  and to project the proximity induced singlet pairing onto that band [15, 17]. In this limit, the low-energy theory of the ring model with flux  $\Phi$  can be mapped onto Kitaev's model [18] with periodic BC and flux  $\Phi + \Phi_0/2$ . The projected model contains doublets  $\{|p\rangle, |-p\rangle\}$  for  $\Phi/\Phi_0 \in [n-1/4, n+1/4]$  with integer  $n$ , whereas for  $\Phi/\Phi_0 \in [n+1/4, n+3/4]$ , the doublet for  $p = 0$  reduces to the singlet  $|p = 0\rangle$ .

In analogy to the generalized variational approach in Ref. [13], we consider variational WFs for the projected Hamiltonian. For each doublet, states with even (+) and odd (−) parity are generated by applying the operators

$$P_-(p) = s_p c_p^\dagger + t_p c_{-p}^\dagger, \quad (6a)$$

$$P_+(p) = u_p + v_p c_p^\dagger c_{-p}^\dagger \quad (6b)$$

to the vacuum state. Here the  $c$  operators denote electrons of the lower helical band  $\epsilon_-(p)$ . General ansatz WFs for even (odd) parity are

$$|\Psi_{e(o)}\{\tau_p\}\rangle = \prod_{p \geq 0} P_{\tau_p}(p)|0\rangle, \quad \prod \tau_p = +1(-1), \quad (7)$$

where  $|0\rangle$  is the vacuum for the  $c$  electrons. To obtain the energy spectrum for arbitrary magnetic flux, we first

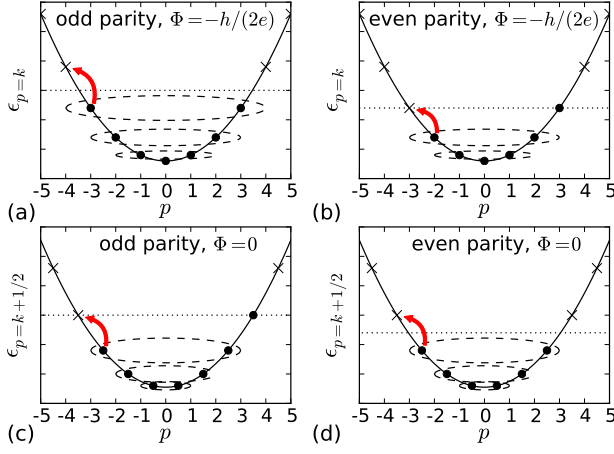


FIG. 2. (Color online) Sketch of the dispersion and the effective pairing for the lower helical band  $\epsilon_-(p)$ . The  $\circ$ -markers (x) denote the occupied (empty) single-particle levels for  $\Delta = 0$ . The dashed ellipses illustrate the paired single-particle levels when switching on the proximity induced SC pairing potential. The red arrows indicate the transport of a single quasiparticle to produce the lowest excited state.

minimize the Ginzburg-Landau free energy Eq. (4) to find the pair wavenumber  $q$ , which is then used to construct the grand canonical mean-field ansatz WFs Eq. (7) with  $p = k - q/2 + 1/2$ . For each set of  $\{\tau_p\}$ , we determine the corresponding energy by unbiased minimization of  $E(N, \{\tau_p\}) = \langle H \rangle + \mu_N N$  with respect to the variational parameters  $(s_p, \dots, v_p)$ . Here  $\mu_N$  is fixed by the mean particle number  $N = \langle \sum_p c_p^\dagger c_p \rangle$  in the SM nanowire. By rank-ordering the  $E(N, \{\tau_p\})$ , we find the ground states for both even and odd parity as function of mean particle number. To obtain the excited states, we then apply the Bogoliubov operators  $a_{p,1}^\dagger = u_p c_p^\dagger - v_p c_{-p}$  and  $a_{p,2}^\dagger = u_p c_{-p}^\dagger + v_p c_p$  with  $p > 0$  to the ground state WF.

In figures 2(a) and 2(b) we sketch a bare parabolic dispersion and the generalized time-reversed partners for  $\Phi = -h/2e$ . The ground-state WF for odd parity is given by  $|\Psi_o^{\text{gs}}\rangle = P_-(0) \prod P_+(p)|0\rangle$ , where all time-reversed partners are paired and the zero momentum electron is unpaired. The lowest excited state is given by two broken pairs at  $p_F$  and  $p_F + 1$ ,  $|\Psi_o^{ij}\rangle = a_{p_F,i}^\dagger a_{p_F+1,j}^\dagger |\Psi_o^{\text{gs}}\rangle$ , which shows up in a spectroscopic gap of  $2\Delta_{\text{eff}}$ . On the other hand, the ground-state for even parity is given by  $|\Psi_e^{\text{gs}}\rangle = P_-(0) P_-(p_F) \prod P_+(p)|0\rangle$  with two unpaired electrons. In contrast to the odd parity case, we find the lowest excited state by breaking the pair at  $p_F - 1$  and creating a new one at  $p_F$ ,  $|\Psi_e^{ij}\rangle = a_{p_F,i}^\dagger a_{p_F-1,j}^\dagger |\Psi_e^{\text{gs}}\rangle$ . Therefore, the excitation energies for the even parity are given by the level spacing. In figures 2(c) and 2(d) we illustrate the pairing for  $\Phi = 0$ . Here, we find that the behavior is reversed compared to the case  $\Phi = -h/2e$ , i.e. the ground state for the even parity contains only paired levels whereas the ground state for the odd parity has one unpaired electron at the Fermi surface.

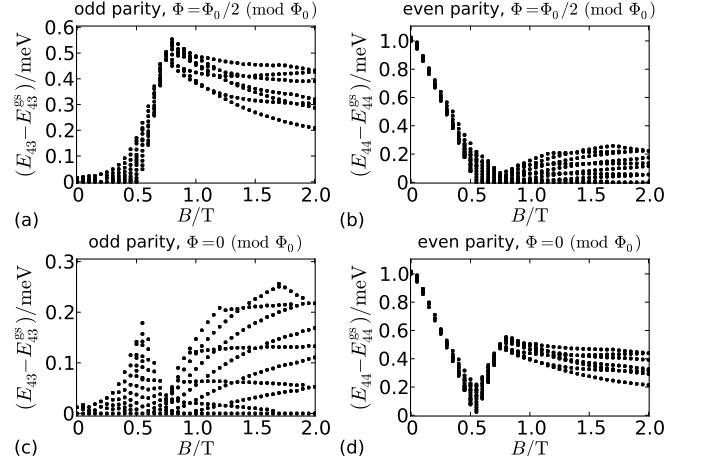


FIG. 3. Magnetic field dependence of the energy differences  $\delta E(N)$ .  $B$  is varied in discrete steps with the flux always being a (half-) integer multiple of  $\Phi_0$ .

*Numerical results.*—We now consider the full Hilbert space again. In analogy to what we explained above, we define generalized operators  $P_\pm(k)$  for each quadruplet of the unprojected Hamiltonian and construct the ansatz WFs as in Eq. (7). We then minimize the energy  $E(N) = \langle H \rangle + \mu_N N$ , where  $N = \langle \sum_{k\sigma} \psi_{k\sigma}^\dagger \psi_{k\sigma} \rangle$ , to obtain the ground state [13]. The lowest excited states are again given by pairwise creation of Bogoliubov quasiparticles near the Fermi surface. We note that in the coupled SM/SC system, the number of electrons in the nanowire is not a good quantum number and the use of grand canonical WFs is fully justified [13]. We have verified that the excitation spectrum depends smoothly on the mean particle number  $N$ .

It has been proposed that InAs and InSb are suitable materials for the nanowires due to their strong spin-orbit coupling [15, 16]. For  $R = 0.5 \mu\text{m}$ , characteristic values for these SM materials are  $\hbar^2/(2m^*R^2) = 0.002 \text{ meV}$ ,  $g\mu_B = 2 \text{ meV/T}$ , and  $\alpha/R = 0.02 \text{ meV}$  [21]. Furthermore, we consider a proximity potential  $\Delta = 0.5 \text{ meV}$  which leads for  $E_Z = 1 \text{ meV}$  and  $\mu = 0$  to an effective pairing gap of  $\Delta_{\text{eff}}(k_F) \approx 0.2 \text{ meV}$ . To ensure single-electron tunneling through the SM/SC-system, we consider the case  $E_C \gg \Delta_{\text{eff}}(k_F)$ .

The external magnetic field can be used to drive the hybrid system through the topological phase transition. In the following,  $B$  is varied in discrete steps with the flux always being a (half-) integer multiple of  $\Phi_0$ , such that the only effect of the variation in  $B$  is a variation of the Zeeman energy. Figure 3 shows the energy differences  $\delta E(N)$  between the excited states and the ground state as functions of the magnetic field for several combinations of magnetic flux and parity. We see qualitative differences between the topological trivial phase of the nanowire for  $B \lesssim 0.5 \text{ T}$  and the nontrivial phase for  $B \gtrsim 0.5 \text{ T}$  [10]. In the topological trivial phase, we find

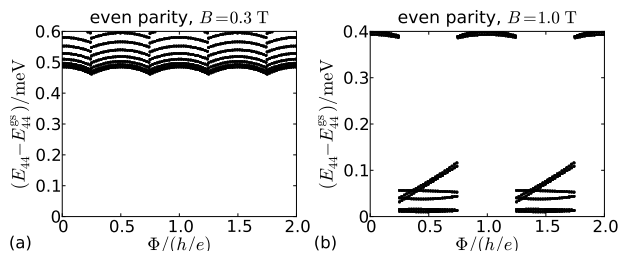


FIG. 4. Energy differences  $\delta E(N)$  as function of the magnetic flux, a) for  $B = 0.3$  T, and b) for  $B = 1.0$  T. Not all higher energies are shown.

signatures that are typical results for SC in ultrasmall metallic grains [12, 13]: if the electron parity in the system is even, the excitation spectrum displays a large spectroscopic gap  $\sim 2\Delta$ , whereas no such gap appears for an odd electron parity. This parity effect is independent of the magnetic flux. The origin of the spectroscopic gap for even parity is that all excited states involve at least two unpaired electrons, while for odd parity the ground state has one unpaired electron and therefore the lowest excitation energies are given by the level spacing near the Fermi surface.

For  $B \gtrsim 0.5$  T we observe a strikingly different parity effect. In stark contrast to the trivial phase, we find that the excitation energies depend on both magnetic flux and electron parity, as illustrated in Fig. 2. In figures 3(a) and 3(d) we find a spectroscopic gap that originates from breaking two pairs, which costs at least  $2\Delta_{\text{eff}}(k_F)$ . In contrast, the excitation energies in Figs. 3(b) and 3(c) are given by the level spacing at the Fermi energy. The topological phase transition is mirrored by the closing and reopening of the excitation gap in Fig. 3(d).

In figure 4 we show the energy differences  $\delta E(N)$  as function of the magnetic flux for both the topological trivial ( $B = 0.3$  T) and nontrivial sectors ( $B = 1.0$  T) for even parity. As shown in Fig. 4(a), in the topologically trivial phase the excitation energies for even parity show a  $\Phi_0/2$  flux periodic gap. For the odd parity case we have verified that the excitation energies are determined by the level spacing. In stark contrast to the trivial phase, we find a  $\Phi_0$  periodic spectrum of the nontrivial phase in Fig. 4(b). For even parity, the excitation energies for  $\Phi/\Phi_0 \in (1/4, 3/4)$  are given by the level spacing at the Fermi surface, while they display the effective gap  $2\Delta_{\text{eff}}(k_F)$  for  $\Phi/\Phi_0 \in (3/4, 5/4)$  due to the pairwise creation of Bogoliubov quasiparticles. For the odd parity case we qualitatively find the same spectrum but shifted by  $\Phi_0/2$ , as follows from the earlier discussion.

We now want to relate the  $\Phi_0$  flux periodicity in the topologically nontrivial phase to the recently discovered  $4\pi$ -periodicity of the Josephson current between two TSCs [4, 18]. To leading order in the tunnel coupling, the

Josephson energy between two 1D TSCs is given by

$$H_J(\Delta\phi) = i\gamma_1\gamma_2\Gamma \cos(\Delta\phi/2), \quad (8)$$

where  $\gamma_1, \gamma_2$  are the operators for the Majorana states localized at the ends of the two TSCs connected by the junction,  $\Gamma$  is the tunneling amplitude, and  $\Delta\phi$  the phase difference between the two SCs. The operator  $i\gamma_1\gamma_2$  with eigenvalues  $\pm 1$  describes the parity of the neutral fermion state which is shared between the two Majoranas. For a fixed parity,  $H_J$  has a period of  $4\pi$ . When inserting the Josephson junction into a ring structure, the phase difference between the two ends can be related to the flux through the ring by observing that inserting a flux  $\Phi_0/2$  gives rise to a phase difference of  $2\pi$  across the junction. For this reason, the  $4\pi$  phase periodicity is equivalent to a  $\Phi_0$  flux periodicity of the ring with fixed parity. If the parity is not fixed, a change of  $\Delta\phi$  by  $2\pi \sim \Phi_0/2$  will change the parity of the ground state and the occupancy of the neutral fermion  $(i\gamma_1\gamma_2 + 1)/2$ . This is in full analogy to our finding that in the topologically nontrivial phase the parity of the ground state changes (if coupled to a reservoir) when changing the flux through the ring by  $\Phi_0/2$ . From this analogy we can understand what the physical interpretation of the neutral fermion formed by the Majorana operators  $\gamma_1, \gamma_2$  is: it corresponds to a change in the parity of the total WF, which in turn is independent of the average number of electrons. Since occupying the neutral fermion describes a change in the parity of the pairing WF and not in the mean number of (charged) particles, the term “neutral fermion” is appropriate. Thus the  $4\pi$  periodicity of the Josephson current is intricately related to the  $\Phi_0$  flux periodicity of a spinless  $p + ip$  TSC on a ring.

*Conclusion.*—We have investigated the signatures of Coulomb blocked transport through a SM/SC hybrid nanoring. We have shown that peculiar parity effects in the excitation spectrum allow to distinguish between the topological trivial and nontrivial phases. Furthermore, we have described a clear signature of the topological phase transition, which can be tuned with the help of the external magnetic field. In addition, we have demonstrated that the  $h/e$  flux periodicity of the excitation spectrum in the topological nontrivial phase is reflected in the  $4\pi$  periodicity of the Josephson Hamiltonian for a tunnel junction between two 1D  $p + ip$  TSCs.

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